# ON THE DISPERSION OF GASES UNDER THE ACTION OF THE MEDIUM

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Some general regularities of dispersion of a gas emerging from a nozzle submerged in a liquid are considered. A condition for establishment of the so-called "maximum dispersion" state is formulated.

Pulp aeration is effected by mechanical and pneumatic gas dispersion methods, as well as by means of the gases released from solution upon lowering the pressure [1].

The pneumatic gas dispersion method is most convenient because of its technological simplicity. However, since the bubbles coming out of the holes of a porous vessel tend to merge rapidly (this is facilitated by the differing diameters of the bubbles and by the close spacing of the holes), this method has not yet come into practical use.

There has been increased interest lately in the study of the dispersion of a gas emerging from a single nozzle. Several authors [2, 3] have considered states associated with small excess pressures such that single bubbles emerge from the nozzle.

The present study concerns states associated with higher pressures which produce larger number of bubbles in a system.

It is clear that the establishment of a given gas dispersion state depends on the following quantities: 1) the density and viscosity of the liquid; 2) the surface tension at the liquid-gas boundary; 3) the radii of the nozzle and bubbles; 4) the viscosity of the gas; 5) the height of the liquid column; 6) the atmospheric and gas pressure; 7) the surface tension at the liquid-solid-gas boundary. As regards the gas density, it is clearly determined by the pressure.

The surface tension at the gas-solid and solid-liquid must in our case appear as the difference between these two quantities, since the solid-liquid boundary is replaced by the solid-gas boundary during passage of the gas. Hence, instead of two surface tensions we have the expression [4] $\sigma_0 \cos\Theta$.

The height of the liquid column and the atmospheric pressure determine the external pressure. The final expression must include the excess pressure given by the expression

$$\Delta p = p - (p_{at} + \rho g h).$$

Hence, the dimensionless criterion must be a combination of the following mutually independent quantities: the liquid density, the liquid and gas viscosities, the surface tension at the liquid-gas boundary, the radii of the bubble and the nozzle hole, and the excess gas pressure. Each of the following pairs of quantities: liquid and gas viscosities, nozzle and bubble radii, and $\sigma_0$ and $\sigma_0 \cos\Theta$ are in the same units.

Hence, the condition of dimensionlessness must be sought for a set of five quantities measured in units of viscosity, surface tension, pressure, density, and length.

Since the total number of basic units of measurement for the given set of quantities is three, it follows by the $\pi$-theorem that the number of dimensionless criteria determining a given dispersion state must be two. Hence, the system of five equations can be solved for any two* undetermined coefficients. We then have

$$k = \left(\frac{\sigma^2 \rho}{\eta^2 \Delta p}\right)^{\zeta} \left(\frac{\Delta p r}{\sigma}\right)^{\xi}. \tag{1}$$

The dimensionlessness of k in the above expression is guaranteed for all $\zeta$ and $\xi$. In order for the quantity k to reflect the meaning of the problem, $\zeta$ and $\xi$ or at least one of them must be different from zero. Let us assume that $\zeta = \xi = 1$. This yields

$$F = \frac{\Delta p r}{\sigma}, \tag{2}$$

$$Q = \frac{\rho}{\Delta p}\left(\frac{\sigma}{\eta}\right)^2. \tag{3}$$

The first of these expressions relates quantities characterized by linear dimensions (the bubble or nozzle hole radius) to the excess pressure and surface tension. In the special case where F = 1 and r is the nozzle radius only, expression (2) becomes the familiar Laplace equation. Here $\Delta p$ is the minimum excess pressure required in order for air to pass through the nozzle.

Moreover, if we recall that according to expression (2) the sum of exponents of the bubble and nozzle radii is equal to unity, then this expression becomes the Karabanov–Frumkin formula which also contains $\Delta p$, $\sigma$, and the bubble and nozzle radii.

Expression (3), which consists of the liquid and gas parameters only, must characterize the dispersion states for a given nozzle. This follows from the obvious fact that a given gas emergence state (including the "flame" state) for the same nozzle arises with different excess pressures.

Hence, regardless of the parameters of the gas and fluid, every nozzle or hole from which the gas emerges is characterized by two dimensionless criteria (F and Q) whose values change from state to state. The

*Since we are dealing with a combination of five elements in pairs, it follows that in general we must have ten characteristic quantities specifying a given state. However, only two of the quantities turn out to be nonidentical by virtue of the $\pi$-theorem.

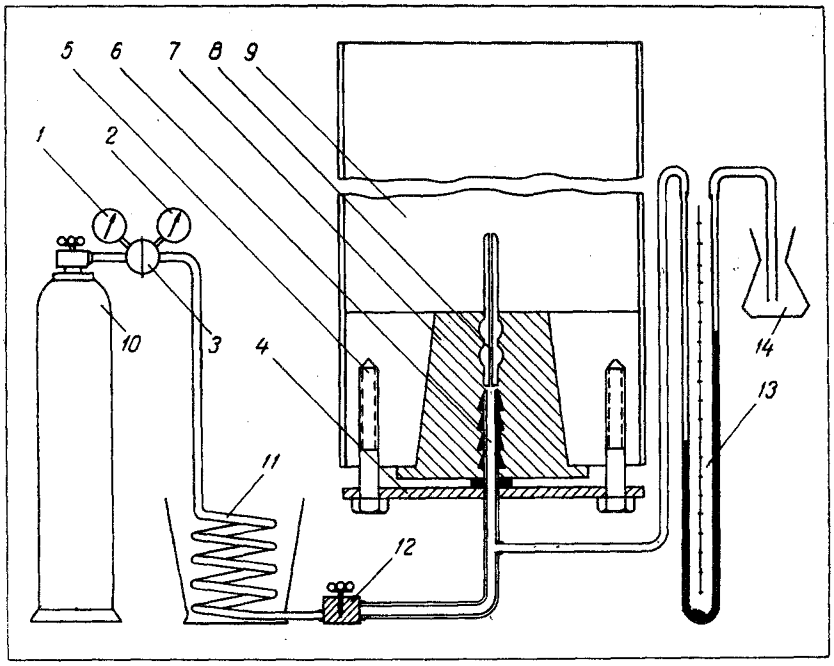


Fig. 1. Diagram of the experimental apparatus: 1) 125-tech. atm. pressure gauge; 2) 25-tech. atm. pressure gauge; 3) reducer; 4) flange which presses connecting pipe 6 and plug 7 to the bottom of column 9 by means of bolts 5; 8) molybdenum glass capillary; 10) compressed air carboy; 11) refrigerator coil for cooling the air entering the column; 12) needle valve; 13) mercury pressure gauge; 15) water-filled vessel (a safety device in case of ejection of the mercury from the pressure gauge).

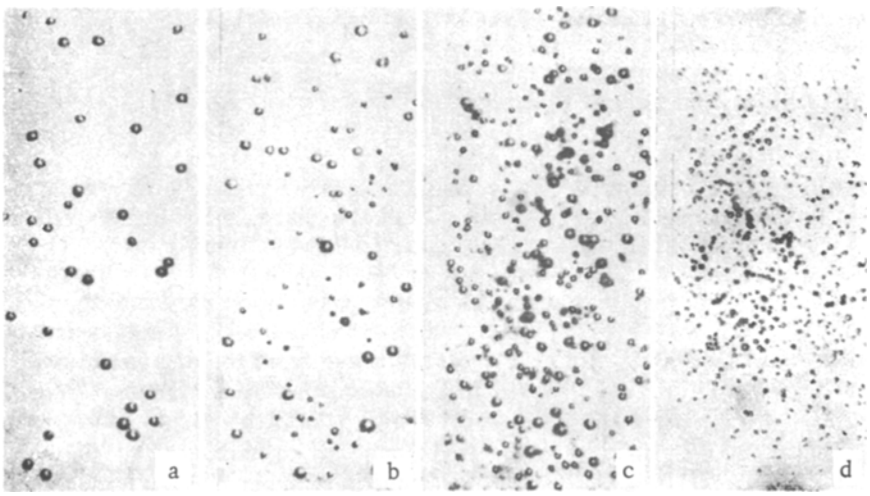


Fig. 2. Characteristic pictures of gas dispersion under the action of the medium for a nozzle with a hole diameter of 125 $\mu$.

criterion Q indicates the corresponding state, while F serves to determine the bubble diameter.

The physical significance of formula (2) is evident from the fact that it becomes the familiar equation expressing the equality of the gas pressures inside and outside the individual bubbles. This condition applies equally to a single bubble formed at the outlet of a nozzle and to bubbles formed as a result of the fragmentation of a jet or of larger bubbles.

In order to investigate the physical meaning of expression (3) we rewrite it as

$$Q = \left(\frac{\sigma}{\eta}\right)^2 \rho \frac{1}{\Delta p}. \tag{4}$$

The denominator Δp is the pressure which promotes growth of the bubbles, expansion of the jet, and scattering of the bubbles in the medium, i.e., it is an external factor. The numerator is the force with which the mass of fluid contained in a unit volume resists the above processes, i.e., acts in the opposite direction. Since all the quantities appearing in the numerator are parameters of the media in contact, this force characterizes the resistance of the system to bubble growth or gas dispersion.

The square of the ratio $\sigma/\eta$ is due to the fact that we are dealing with a force per unit area (as in the case of the excess pressure). Hence, a given dispersion state is established for a specific ratio of these two opposing forces.

In computing Q on the basis of experimental data it is necessary to bear in mind the fact that $\eta$ includes not only the fluid viscosity, but also the viscosity of the gas passing through the nozzle, while $\sigma$ includes not only the coefficient of surface tension at the liquid-gas boundary $\sigma_0$, but also the difference between the coefficients of surface tension at the liquid-solid and gas-solid boundaries, this difference being the product $\sigma_0 \cos\Theta$, where $\Theta$ is the contact angle of wetting of the solid by the liquid. Hence,

$$Q = \frac{\sigma_0^{\beta} (\sigma_0 \cos\Theta)^{\beta_1} \rho}{\eta_1^{\alpha} \eta_g^{\alpha_1} \Delta p}. \tag{5}$$

Recalling that

$$\alpha + \alpha_1 = 2, \quad \beta + \beta_1 = 2, \tag{6}$$

we have

$$Q = \frac{\sigma_0^2 (\cos\Theta)^{2-\beta} \rho}{\eta_1^{\alpha} \eta_g^{2-\alpha} \Delta p} \tag{7}$$

or

$$\log Q = \alpha \log \frac{\eta_g}{\eta_1} + \beta \log \frac{1}{\cos\Theta} + z, \tag{8}$$

where

$$z = 2 \log \frac{\sigma_0 \cos\Theta}{\eta_g} + \log \frac{\rho}{\Delta p}. \tag{9}$$

Expression (8) contains three unknowns, Q, $\alpha$, and $\beta$; their values must change from state to state. We were interested in the values of these unknowns which characterized maximum dispersion for a nozzle 125 μ in diameter. For this purpose we employed the relatively simple setup shown in Fig. 1. The gas was air and the liquids were sugar solutions of water with sugar concentrations of 1:10, 1:3, and 2:3 and four batches of transformer oil dissolved in gasoline.

We began by determining all of the characteristic states for each liquid and finding the appropriate values of the excess air pressure.

The states noted in our experiments were as follows:

1. Gas emerged in single bubbles for a low excess pressure determined from the familiar Laplace equation.

2. The frequency of bubble breakaway increased with rising pressure until the bubbles formed a chain. The bubbles were markedly smaller than in the first state.

3. The bubbles formed by the more compressed gas then increased rapidly in volume until equilibrium between the internal and external pressures was achieved (Fig. 2a). The bubble diameter was much larger than in the second state.

4. With further increases in excess pressure the chain of gas bubbles became shorter and a single bubble formed at the tip of the slender gas jet. Small bubbles formed at the surface of this single bubble. As a result, bubbles of two different sizes traveled through the medium (Fig. 2b).

5. At some value of the excess pressure, the growth of the small bubbles and the shrinkage of the large ones (see description of the fourth state) produced the state of maximum gas dispersion (Fig. 2c). Comparison indicates that this was the case where the system was maximally monodisperse and saturated with small bubbles.

6. The next state, which was essentially preserved at high pressures (8 atm in our experiments), differed from the fourth state in the sizes of both the large and small bubbles (Fig. 2c), i.e., the system was distinctly polydisperse.

The figure shows successive pictures of dispersion of a gas emerging from small holes.

Dispersion in the case of nozzles with large-diameter openings differs from that described in that the jet forms a "flame" beginning with the fourth state. The diameter of the transverse cross section of the flame is always larger than the diameter of the bubble formed at the liquid-gas boundary. Thus, the bubbles formed from the core and surface of the flame cannot be of equal size at any gas pressure, no matter how high. It is true that the self-fragmentation of the large bubbles formed out of the flame core greatly increase the gas dispersion when they reach critical size. Even in this case, however, the secondary bubbles formed as a result of the fragmentation of the critical bubbles were several (as much as ten) times larger than the surface bubbles, since the diameter of the fragmentation bubbles was about two times as large as that of the critical oblate bubbles.

It is necessary to note that the principal role in the production of bubbles during flame formation is played by the surface forces at the gas-liquid boundary.

In our case the small diameter of the nozzle hole resulted in a very slender jet in the liquid prior to fragmentation; the velocity of the gas stream in the jet was considerable. For this reason a major role was played by the inertial forces which generally determine the depth of penetration of the jet into the medium. In our experiments, in which the excess gas pressure equaled not hundreds or thousands of atmospheres, but only 1.0–2.0 atm, the relative difference in pressure in the fourth and local states did not exceed 20–30%, and the jet height remained almost constant. Thus, the prevalence of a given state is not determined by the jet height.

The bubbles formed at the end of the jet cannot be regarded as resulting solely from the fragmentation of the critically large bubbles, since, as computations indicate, the sum diameter of all the bubbles at the end of the jet (until they come apart as they rise) is several times larger than that of the critical bubble alone.

The phenomenon of gas dispersion under the action of the medium is, in our view, analogous to the impact of a jet against a vibrating plate in which the character of flame fragmentation is determined by the stream velocity and the vibration frequency of the plate. The optimal gas dispersion state is then one which is characterized by a specific relationship between the gas stream velocity and the frequency of bubble breakaway from the tip of the gas jet. The role of the plate is played by the boundary between the liquid and the fan-shaped end of the jet: the vibration factor created by bubble breakaway.

The characteristic states, while they remain the same for different liquids and gases, vary with changes in nozzle diameter. This is fully consistent with the fact that each of the similar dispersion states for a nozzle of any diameter is described by two dimensionless criteria. Hence, the central problem in the study of gas dispersion under the action of the medium is that of determining the criteria Q and F as functions of nozzle radius.

Theoretical solution of this problem is a fairly complicated matter. It will be solved eventually by generalizing experimental data, once a sufficient body of these has been accumulated. We confined ourselves to the problem of establishing the values of the dimensionless criterion Q for the optimal dispersion state for nozzle hole diameters of 125 $\mu$. To this end we used expression (8) to construct a system of seven equations with three unknowns. After elimination of log Q, the system consisted of six equations. Application of the method of least squares to this system yielded two normal equations which we used to determine $\alpha$ and $\beta$. Owing to the simplicity of the method and lack of space we shall merely cite the final results of the computations; it turned out that $\alpha = -1.78$ and $\beta = -2.78$.

Recalling expressions (6), we obtain

$$Q = \frac{\sigma^2 \eta_l^{1.78} \rho (\cos \Theta)^{4.78}}{\eta_g^{3.78} [p - (p_{at} + \rho gh)]}. \quad (10)$$

For a nozzle of diameter 125 $\mu$ the value of Q turned out to be $4.48 \cdot 10^8$.

The deviations of the Q values for most of the experiments, i.e., for five cases, did not exceed 25%. For two experiments with small surface tension at the liquid-air boundary the deviations amounted to 50% of the Q values. However, if we recall that the Reynolds number, which includes only four quantities, varies from 1200 to 2000, then our value of Q can be regarded as sufficiently accurate for practical purposes.

## NOTATION

$\sigma_0$ is the coefficient of surface tension at the liquid-gas boundary; $\Theta$ is the contact angle of wetting of the nozzle material surface by the liquid; $p_{at}$ is the atmospheric pressure; p is the air pressure; $\rho$ is the density of the liquid; g is gravitational acceleration; h is the height of the liquid column; $\eta_l$ and $\eta_g$ are the dynamic viscosity coefficients of the liquid and gas, respectively; R and r are the radii of the bubble and nozzle, respectively; Q and F are dimensionless criteria; $\alpha$, $\beta$, $\gamma$, $\zeta$, and $\xi$ are the undetermined coefficients; $\pi$ is the ratio of the circumference of a circle to its diameter.

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